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Reliable H_{∞} filter design for sampled-data systems with consideration of probabilistic sensor signal distortion

Zhou Gu^{1,2}, Engang Tian³, Jinliang Liu⁴

 ¹College of Mechanical and Electronic Engineering, Nanjing Forestry University, Nanjing 210037, People's Republic of China
 ²School of Automation, Southeast University, Nanjing 210096, People's Republic of China
 ³Institute of Information and Control Engineering Technology, Nanjing Normal University, Nanjing 210042, People's Republic of China
 ⁴Department of Applied Mathematics, Nanjing University of Finance and Economics, Nanjing, Jiangsu 210046, People's Republic of China
 E-mail: gzh1808@163.com

Abstract: This study is concerned with the reliable filtering problem for the sampled-data system subject to a class of probabilistic sensor signals distortion. A new distortion model is developed by introducing a diagonal random matrix whose elements obey the Gaussian distribution. The main purpose in this study is to design a filter such that the error dynamics of the filtering process subject to the probabilistic sensor signal distortion is mean-square asymptotically stable. Based on the modified delay-central-point (DCP) method and the convexity property of the matrix inequality, new criteria are derived for the existence of the desired H_{∞} filters, by which it leads to much less conservative analysis results. Simulation results are provided to illustrate the effectiveness of the proposed method.

1 Introduction

The filtering problem of sampled-data systems has been attracting considerable research interests over the past decades with a fast development of computer technology [1-9]. In traditional studies, most of the researches focus on single-rate digital systems. Filter design for the linear sampled-data system was concerned in [10, 11], and the problem of filter design for non-linear systems was studied in [12, 13]. In [14-19], Wang and co-workers dealt with the robust filtering problem for uncertain stochastic systems. However, it should be mentioned that in real systems A/D and D/A converters often work at different sampling rates because of various computational load and external disturbance etc.. Therefore another approach that is time-varying sampling period is aroused, see, for example, [6, 8, 9] and the references therein.

It should be noted that the aforementioned results are based on an assumption that sensors operate without any flaws, that is, the filter receives the value of the process accurately. However, the distortion of the sensor usually happens in practice because of some internal or external reasons, such as, the aging of the components, the external disturbance etc. To the best of our knowledge, the design of the sampled-data filter subject to stochastic sensor distortion is still an open problem, although there are some results on filter design considering the abnormal sensor transmission cases [20, 21]. The problem of reliable guaranteed variance filtering against sensor failures is addressed in [20], where the scale-factor of sensor failures belongs to an interval. The fault model of the sensor, however, has its limitations as it cannot cover the practical case. In [21], the filtering problem for a class of discrete-time with consideration of the missing measurement by a random variable satisfying a certain probabilistic distribution on the interval [0, 1] is concerned, however, the sensor output signal is not always missing; sometimes it may fluctuate around the true or a certain value.

This paper is concerned with the problem of H_{∞} reliable filtering of the sampled-data system subject to a probabilistic sensor signal distortion. The main contributions of the obtained results are as follows: (i) a probabilistic sensor signal distortion model for continuous-time system is developed; (ii) the conservativeness of the derived H_{∞} performance analysis result is further reduced by combining the delay-central-point (DCP) method [22] and the nature of convexity property of the matrix inequality. Numerical examples are given to show the effectiveness of the proposed method.

Notation: \mathbb{R}^n denotes the *n*-dimensional Euclidean space, $\mathbb{R}^{n \times m}$ is the set of real $n \times m$ matrices, *I* is the identity matrix of appropriate dimensions, $\|\cdot\|$ stands for the Euclidean vector norm or spectral norm as appropriate. The

notation X > 0 (respectively, X < 0), for $X \in \mathbb{R}^{n \times n}$ means that the matrix X is a real symmetric positive definite (respectively, negative definite). When x is a stochastic variable, $\mathcal{E}{x}$ stands for the expectation of x. The asterisk * in a matrix is used to denote the term that is induced by symmetry. Matrices, if they are not explicitly stated, are assumed to have compatible dimensions.

2 Problem formulation

Consider the controlled process is an linear time invariant (LTI) system

$$\begin{cases} \dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\omega(t) \\ \mathbf{y}(t) = C\mathbf{x}(t) \\ \mathbf{z}(t) = L\mathbf{x}(t) \end{cases}$$
(1)

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the state vector, $\mathbf{y}(t) \in \mathbb{R}^q$ is the measurable output vector, $\mathbf{z}(t)$ is a signal to be estimated, the process noise $\omega(t) \in \mathbb{R}^d$ including model uncertainties and external plant disturbance belongs to $l_2[0 \infty]$. *A*, *B*, *C* and *L* are constant matrices with appropriate dimensions.

As shown in Fig. 1, the filter to be designed is based on the sampled-data mode. The measurement noise cannot be avoided, although measuring instrument is with high accuracy. It leads to the sensor signal distortion as

$$\mathbf{y}_m(t) = \mathbf{\Xi} \mathbf{y}(t) \tag{2}$$

where $y_m(t)$ is an actual measurement of the process output; $\Xi = \text{diag}\{\xi_1, \ldots, \xi_m\}$ is a diagonal matrix, which indicates the rate of signal distortion in every measurement channels; The elements $\xi_i\{\xi_i|0 \le \xi_i \le \sigma, i = 1, \ldots, m\}$ are *m* unrelated variables. Here, we define $\mathcal{E}\{\xi_i\} = \mu_i$ and $\mathcal{E}\{\Xi\} = \overline{\Xi}$.

In this paper, we are interested in obtaining the estimate of the signal z(t) from the actual measured output. The full-order filter to be considered is given as follows

$$\begin{cases} \dot{\mathbf{x}}_{\rm f}(t) = \mathbf{A}_{\rm f} \mathbf{x}_{\rm f}(t) + \mathbf{B}_{\rm f} \mathbf{y}_m(t_k) & t_k \le t < t_{k+1}, \quad k = 0, \ 1, \ 2, \ \dots \\ \mathbf{z}_{\rm f}(t) = \mathbf{C}_{\rm f} \mathbf{x}_{\rm f}(t) \end{cases}$$
(3)

where $x_f(t) \in \mathbb{R}^n$ is the state vector of filter, A_f , B_f and C_f are the filter parameters with appropriate dimensions to be determined. t_k denotes the sampling sequence.

Remark 1: in [20], the sensor failure was considered in the process of the filter design, however, the scale-factor of the sensor failure ξ_i does not include the failure statistical information. In fact, the sensor fault model in [20] are only two special cases modelled in (2). In [23], the authors develop a fault model to describe an intermittent measurement case by using the Bernoulli distribution, however, it cannot cover the case of signal distortion.



Fig. 1 Sampled-data system

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Remark 2: we borrow and extend the idea in [21] to develop the model of signal distortion in every measuring channels. In [21], the authors mainly addressed the measurement missing of the sensors for discrete-time systems by introducing a random variable. Moreover, the authors assumed that the random variables ξ_i takes value in the interval [0, 1], as a matter of fact, the measured signal may be bigger than the real value because of the measurement noise, which is a common occurrence in practical systems, such as, offset thermal drift; however, it has not caused a considerate attention up to now.

It can be shown from (3) and Fig. 1 that the input of the filter holds a constant value till the next sampling instant. Then, we denote $\tau(t) = t - t_k$ for $t_k \le t < t_{k+1}$. It is clear that

$$0 < \underline{h} \le \tau(t) < t_{k+1} - t_k < \bar{h} \tag{4}$$

where \underline{h} and \overline{h} are the lower and upper bounds of distance of two sampling instants, respectively.

Defining $\zeta(t) = \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{x}_{f}(t) \end{bmatrix}$ and $\mathbf{e}(t) = \mathbf{z}(t) - \mathbf{z}_{f}(t)$. We can obtain the following filtering-error system

$$\begin{cases} \dot{\zeta}(t) = \bar{A}\zeta(t) + (\bar{A}_d + \bar{A}_{dr})H\zeta(t - \tau(t)) + \bar{B}\omega(t) \\ e(t) = \bar{L}\zeta(t) \end{cases}$$
(5)

where

$$\bar{A} = \begin{bmatrix} A & 0 \\ 0 & A_{\rm f} \end{bmatrix}, \ \bar{A}_d = \begin{bmatrix} 0 \\ B_{\rm f}\bar{\Xi}C \end{bmatrix}, \ \bar{A}_{dr} = \begin{bmatrix} 0 \\ B_{\rm f}(\Xi - \bar{\Xi})C \end{bmatrix}$$
$$\bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \ \bar{L} = \begin{bmatrix} L & -C_{\rm f} \end{bmatrix}, \ H = \begin{bmatrix} I & 0 \end{bmatrix}$$

3 Main results

In this section, we propose an linear matrix inequation (LMI) approach to solve the H_{∞} filtering problem formulated in the previous section. We first give the following lemma and definitions which will be used in the subsequent development.

Lemma 1: [24] For any constant matrix $R \in \mathbb{R}^{n \times n}$, R > 0, scalars $\overline{\tau}_1 \leq \tau(t) \leq \overline{\tau}_2$, and vector function $\dot{x}: [-\overline{\tau}_2, -\overline{\tau}_1] \rightarrow \mathbb{R}^n$ such that the following integration is well defined, it holds that

$$-\left(\bar{\tau}_{2}-\bar{\tau}_{1}\right)\int_{t-\bar{\tau}_{2}}^{t-\bar{\tau}_{1}}\dot{\mathbf{x}}^{\mathrm{T}}(t)\boldsymbol{R}\dot{\mathbf{x}}(t)$$

$$\leq \begin{bmatrix} \mathbf{x}(t-\bar{\tau}_{1})\\ \mathbf{x}(t-\bar{\tau}_{2}) \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} -\mathbf{R} & *\\ \mathbf{R} & -\mathbf{R} \end{bmatrix}\begin{bmatrix} \mathbf{x}(t-\bar{\tau}_{1})\\ \mathbf{x}(t-\bar{\tau}_{2}) \end{bmatrix}$$
(6)

Lemma 2: [25] Suppose M, N and Ω are constant matrices of appropriate dimensions. Then

$$\left(\tau(t) - \bar{\tau}_1\right) M + \left(\bar{\tau}_2 - \tau(t)\right) N + \mathbf{\Omega} < 0 \tag{7}$$

is true for any $\tau(t) \in \begin{bmatrix} \overline{\tau}_1 & \overline{\tau}_2 \end{bmatrix}$ if and only if

$$\left(\bar{\tau}_2 - \bar{\tau}_1\right) M + \mathbf{\Omega} < 0 \tag{8}$$

$$(\bar{\tau}_2 - \bar{\tau}_1)N + \mathbf{\Omega} < 0 \tag{9}$$

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Definition 1: For a given function $V: C_{F_0}^b([-\tau_2, 0], \mathbb{R}^n) \times S$, its infinitesimal operator \mathcal{L} [26] is defined as

$$\mathcal{L}V(\mathbf{x}_{t}) = \lim_{\Delta \to 0^{+}} \frac{1}{\Delta} \left[\mathcal{E} \left(V(\mathbf{x}_{t+\Delta} | \mathbf{x}_{t}) - V(\mathbf{x}_{t}) \right) \right]$$
(10)

Definition 2: System (5) is said to be asymptotically stable in the mean-square sense with an H_{∞} norm bound γ , if the following conditions hold:

1. System (5) with $\omega(t) = 0$ is asymptotically stable in the mean-square sense.

2. Under the assumption of zero initial condition, it satisfies $\mathcal{E}\{\|\mathbf{e}(t)\|_2\} \le \gamma \|\omega(t)\|_2$ for any non-zero $\omega(t) \in l_2[t_0, \infty]$.

For convenience of description, we define $\delta = (\bar{h} - \underline{h})/2$, then $[\underline{h}, \bar{h}] = [h, h_0] \cup [h_0, \bar{h}]$, where $h_0 = \underline{h} + \delta$. The following delay-dependent conditions can be got to guarantee $\mathcal{E}\{\mathcal{L}V(\zeta_t)\} < 0$ by constructing the Lyapunov functional $V(\zeta_t)$ and checking the infinitesimal operator of $V(\zeta_t)$ in both the case of $\tau(t) \in [\underline{h}, h_0]$ and the case of $\tau(t) \in [h_0, \bar{h}]$.

Theorem 1: For given scalars \underline{h} , \overline{h} and γ , if there exist matrices P > 0, $Q_i > 0$, $R_i > 0$ (i = 0, 1, 2), M, N, A_f , B_f and C_f with appropriate dimensions satisfying (11), then the system (5) is mean-square asymptotically stable with H_{∞} norm bound γ .

$$\begin{bmatrix} \Pi_{i} + \Lambda_{i} + \Lambda_{i}^{\mathrm{T}} & * & * & * \\ \mathbf{RA} & -\mathbf{R} & * & * \\ \tilde{\mathbf{L}} & 0 & -\mathbf{I} & * \\ \Upsilon_{ij} & 0 & 0 & -\mathbf{R}_{i} \end{bmatrix} < 0 \ (i, \ j = 1, \ 2)$$

$$(11)$$

where (see equations at the bottom of the page)

$$\Pi_{11} = \boldsymbol{P}\bar{\boldsymbol{A}} + \bar{\boldsymbol{A}}^{T}\boldsymbol{P} + H^{T}(\boldsymbol{Q}_{1} + \boldsymbol{Q}_{2} - \boldsymbol{R}_{0})H$$

$$Y_{11} = \sqrt{\delta}M^{T}, \ Y_{12} = \sqrt{\delta}N^{T}, \ Y_{21} = \sqrt{\delta}S^{T}, \ Y_{22} = \sqrt{\delta}T^{T}$$

$$\Lambda_{1} = [0\boldsymbol{M}N - \boldsymbol{M} - N00], \quad \Lambda_{2} = [00\boldsymbol{S} - \boldsymbol{T}\boldsymbol{T} - \boldsymbol{S}0]$$

$$\mathcal{R} = \underline{h}^{2}\boldsymbol{R}_{1} + \delta\boldsymbol{R}_{1} + \delta\boldsymbol{R}_{2}, \ \tilde{\boldsymbol{L}} = [\bar{\boldsymbol{L}}\ 0\ 0\ 0\ 0\ 0]$$

$$\mathcal{A} = [H\bar{\boldsymbol{A}} \quad 0 \quad H\bar{\boldsymbol{A}}_{d} \quad 0 \quad \boldsymbol{O} \quad \boldsymbol{B}]$$

Proof: choose the Lyapunov function as

$$V(\zeta_t) = V_1(\zeta_t) + V_2(\zeta_t) + V_3(\zeta_t)$$

where

$$V_{1}(\zeta_{t}) = \zeta^{\mathrm{T}}(t)\boldsymbol{P}\zeta(t)$$

$$V_{2}(\zeta_{t}) = \int_{t-\underline{h}}^{t} \zeta^{\mathrm{T}}(s)H^{\mathrm{T}}\boldsymbol{Q}_{1}H\zeta(s) \,\mathrm{d}s + \int_{t-\overline{h}}^{t} \zeta^{\mathrm{T}}(s)H^{\mathrm{T}}\boldsymbol{Q}_{2}H\zeta(s) \,\mathrm{d}s$$

$$+ \int_{t-h_{0}}^{t} \zeta^{\mathrm{T}}(s)H^{\mathrm{T}}\boldsymbol{Q}_{0}H\zeta(s) \,\mathrm{d}s$$

$$V_{3}(\zeta_{t}) = h \int_{-\underline{h}}^{0} \int_{t+s}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(v)\boldsymbol{R}_{0}\dot{\boldsymbol{x}}(v)\mathrm{d}v\mathrm{d}s + \int_{-h_{0}}^{-\underline{h}} \int_{t+s}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(v)\boldsymbol{R}_{1}\dot{\boldsymbol{x}}(v)\mathrm{d}v\mathrm{d}s$$

$$+ \int_{-\overline{h}}^{-h_{0}} \int_{t+s}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(v)\boldsymbol{R}_{2}\dot{\boldsymbol{x}}(v)\mathrm{d}v\mathrm{d}s$$

Use the infinitesimal operator (10) for system (5). It yields

$$\begin{aligned} \mathcal{E}\left\{\mathcal{L}V_{1}\left(\zeta_{t}\right)\right\} &= 2\mathcal{E}\left\{\zeta^{T}(t)\boldsymbol{P}\left[\bar{\boldsymbol{A}}\zeta(t) + \bar{\boldsymbol{A}}_{d}\zeta(t - \tau(t)) + \bar{\boldsymbol{B}}\omega(t)\right]\right\}\\ \mathcal{E}\left\{\mathcal{L}V_{2}\left(\zeta_{t}\right)\right\} &= \mathcal{E}\left\{\zeta^{T}(t)H^{T}\left(\boldsymbol{Q}_{0} + \boldsymbol{Q}_{1} + \boldsymbol{Q}_{2}\right)H\zeta(t)\\ &- \zeta^{T}(t - \bar{\boldsymbol{h}})H^{T}\boldsymbol{Q}_{1}H\zeta(t - \bar{\boldsymbol{h}})\\ &- \zeta^{T}(t - \bar{\boldsymbol{h}})H^{T}\boldsymbol{Q}_{2}H\zeta(t - \bar{\boldsymbol{h}}) \in\\ &- \zeta^{T}(t - h_{0})H^{T}\boldsymbol{Q}_{0}H\zeta(t - h_{0})\right\}\\ \mathcal{E}\left\{\mathcal{L}V_{3}\left(\zeta_{t}\right)\right\} &= \mathcal{E}\left\{\eta^{T}(t)\mathcal{A}^{T}\mathcal{R}\mathcal{A}\eta(t) - h\int_{t-h}^{t}\dot{\boldsymbol{x}}^{T}(s)\boldsymbol{R}_{0}\dot{\boldsymbol{x}}(s)ds\\ &- \int_{t-h_{0}}^{t-h}\dot{\boldsymbol{x}}^{T}(s)\boldsymbol{R}_{1}\dot{\boldsymbol{x}}(s)ds - \int_{t-\bar{h}}^{t-h_{0}}\dot{\boldsymbol{x}}^{T}(s)\boldsymbol{R}_{2}\dot{\boldsymbol{x}}(s)ds\right\}\end{aligned}$$

where

$$\eta(t) = \begin{bmatrix} \zeta^{\mathrm{T}}(t) & \boldsymbol{x}^{\mathrm{T}}(t-\underline{h}) & \boldsymbol{x}^{\mathrm{T}}(t-\tau(t)) \\ & \boldsymbol{x}^{\mathrm{T}}(t-h_0) & \boldsymbol{x}^{\mathrm{T}}(t-\bar{h}) & \boldsymbol{\omega}^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$$

$$\Pi_{1} = \begin{bmatrix} \Pi_{11} & * & * & * & * & * & * \\ \mathbf{R}_{0}H & -\mathbf{Q}_{1} - \mathbf{R}_{0} & * & * & * & * & * \\ \bar{\mathbf{A}}^{\mathsf{T}}\mathbf{P} & 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & -\mathbf{Q}_{0} - \mathbf{R}_{2}/\delta & * & * & * \\ 0 & 0 & 0 & \mathbf{R}_{2}/\delta & -\mathbf{Q}_{2} - \mathbf{R}_{2}/\delta & * \\ \bar{\mathbf{B}}^{\mathsf{T}}P & 0 & 0 & 0 & 0 & -\gamma^{2} \end{bmatrix}$$
$$\Pi_{2} = \begin{bmatrix} \Pi_{11} & * & * & * & * & * \\ \mathbf{R}_{0}H & -\mathbf{Q}_{1} - \mathbf{R}_{0} - \mathbf{R}_{1}/\delta & * & * & * & * \\ \bar{\mathbf{A}}^{\mathsf{T}}\mathbf{P} & 0 & 0 & * & * & * \\ 0 & \mathbf{R}_{1}/\delta & 0 & -\mathbf{Q}_{0} - \mathbf{R}_{1}/\delta & * & * \\ 0 & 0 & 0 & 0 & -\mathbf{Q}_{2} & * \\ \bar{\mathbf{B}}^{\mathsf{T}}\mathbf{P} & 0 & 0 & 0 & 0 & -\gamma^{2} \end{bmatrix}$$

Using Lemma 1, we have

$$-\underline{h} \int_{t-h}^{t} \dot{\mathbf{x}}^{\mathrm{T}}(s) \mathbf{R}_{1} \dot{\mathbf{x}}(s) \, \mathrm{d}s \leq \begin{bmatrix} \boldsymbol{\zeta}(t) \\ \mathbf{x}(t-\underline{h}) \end{bmatrix}^{\mathrm{T}} \\ \begin{bmatrix} -H^{\mathrm{T}} \mathbf{R}_{0} H & * \\ \mathbf{R}_{0} H & -\mathbf{R}_{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\zeta}(t) \\ \mathbf{x}(t-\underline{h}) \end{bmatrix}$$
(12)

Combining the results of $\mathcal{E}\{\mathcal{L}V_i(\zeta_i)\}(i = 1, 2, 3)$, we can obtain

$$\mathcal{E}\left\{\mathcal{L}V\left(\zeta_{t}\right)\right\} \leq \mathcal{E}\left\{2\zeta^{T}(t)\boldsymbol{P}\left[\bar{A}\zeta(t) + \bar{A}_{d}\boldsymbol{x}\left(t - \boldsymbol{\tau}(t)\right) + \bar{\boldsymbol{B}}\omega(t)\right] \\ + \zeta^{T}(t)H^{T}\left(\boldsymbol{Q}_{0} + \boldsymbol{Q}_{1} + \boldsymbol{Q}_{2}\right)H\zeta(t) \\ - \zeta^{T}(t - \underline{h})H^{T}\boldsymbol{Q}_{1}H\zeta(t - \underline{h}) \\ - \zeta^{T}(t - \bar{h})H^{T}\boldsymbol{Q}_{2}H\zeta(t - \bar{h}) \\ - \zeta^{T}(t - h_{0})H^{T}\boldsymbol{Q}_{0}H\zeta(t - h_{0}) \\ + \boldsymbol{\eta}^{T}(t)\mathcal{A}^{T}\mathcal{R}\mathcal{A}\boldsymbol{\eta}(t) \\ + \left[\frac{\zeta(t)}{x(t - \underline{h})}\right]^{T}\left[-H^{T}\boldsymbol{R}_{0}H \quad * \\ \boldsymbol{R}_{0}H \quad -\boldsymbol{R}_{0}\right]\left[\boldsymbol{\zeta}(t) \\ x(t - \underline{h})\right] \\ - \int_{t - h_{0}}^{t - \underline{h}} \dot{\boldsymbol{x}}^{T}(s)\boldsymbol{R}_{1}\dot{\boldsymbol{x}}(s)ds - \int_{t - \bar{h}}^{t - h_{0}} \dot{\boldsymbol{x}}^{T}(s)\boldsymbol{R}_{2}\dot{\boldsymbol{x}}(s)ds \\ + e^{T}(t)e(t) - \gamma^{2}\omega^{T}(t)\omega(t) \\ - e^{T}(t)e(t) + \gamma^{2}\omega^{T}(t)\omega(t)\right\}$$
(13)

As mentioned above, $\tau(t) \in [\underline{h}, h_0]$ or $[h_0, \overline{h}]$ at any instant, we define the following two sets

$$\mathcal{F}_1 = \left\{ t : \tau(t) \in [\underline{h}, \ h_0] \right\} \tag{14}$$

$$\mathcal{F}_2 = \left\{ t: \tau(t) \in [h_0, \ \bar{h}] \right\}$$
(15)

In the following, we will discuss $\mathcal{E}\{\mathcal{L}V(\zeta_t)\}$ for the two cases in relation to $t \in \mathcal{F}_1$ and $t \in \mathcal{F}_2$. *Case 1:* For $t \in \mathcal{F}_1$: Applying Lemma 1, we can obtain deduced

$$-\int_{t-\bar{h}}^{t-h_{0}} \dot{\mathbf{x}}^{\mathrm{T}}(s) \mathbf{R}_{2} \dot{\mathbf{x}}(s) \mathrm{d}s$$

$$\leq \begin{bmatrix} \mathbf{x}(t-h_{0}) \\ \mathbf{x}(t-\bar{h}) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -\mathbf{R}_{2} & * \\ \mathbf{R}_{2} & -\mathbf{R}_{2} \end{bmatrix} \begin{bmatrix} \mathbf{x}(t-h_{0}) \\ \mathbf{x}(t-\bar{h}) \end{bmatrix} \quad (16)$$

By using slack matrices method [27], we have

$$2\eta^{\mathrm{T}}(t)M\left[\mathbf{x}(t-\underline{h})-\mathbf{x}(t-\tau(t))-\int_{t-\tau(t)}^{t-\mathrm{h}}\dot{\mathbf{x}}(s)\mathrm{d}s\right]=0 \quad (17)$$
$$2\eta^{\mathrm{T}}(t)N\left[\mathbf{x}(t-\tau(t))-\mathbf{x}(t-h_{0})-\int_{t-h_{0}}^{t-\tau(t)}\dot{\mathbf{x}}(s)\mathrm{d}s\right]=0 \quad (18)$$

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Note that

$$-2\eta^{\mathrm{T}}(t)\boldsymbol{M} \int_{t-\tau(t)}^{t-\underline{h}} \dot{\boldsymbol{x}}(s) \mathrm{d}s \leq \left(\tau(t) - \underline{h}\right) \eta^{\mathrm{T}}(t) \boldsymbol{M} \boldsymbol{R}_{1}^{-1} \boldsymbol{M}^{\mathrm{T}} \eta(t) \\ + \int_{t-\tau(t)}^{t-\underline{h}} \dot{\boldsymbol{x}}^{\mathrm{T}}(s) \boldsymbol{R}_{1} \dot{\boldsymbol{x}}(s) \mathrm{d}s \tag{19}$$

$$-2\eta^{\mathrm{T}}(t)N\int_{t-h_{0}}^{t-\tau(t)}\dot{\mathbf{x}}(s)\mathrm{d}s \leq (h_{0}-\tau(t))\eta^{\mathrm{T}}(t)NR_{1}^{-1}N^{\mathrm{T}}\eta(t)$$
$$+\int_{t-h_{0}}^{t-\tau(t)}\dot{\mathbf{x}}^{\mathrm{T}}(s)R_{1}\dot{\mathbf{x}}(s)\mathrm{d}s \qquad (20)$$

Combining (13)-(20), we can deduce

$$\mathcal{E}\{\mathcal{L}V(\mathbf{x}_{t})\} \leq \mathcal{E}\left\{\eta^{\mathrm{T}}(t)\left[\Pi_{1} + \mathcal{A}^{\mathrm{T}}\mathcal{R}\mathcal{A} + \Lambda_{1} + \Lambda_{1}^{\mathrm{T}} + \tilde{\boldsymbol{L}}^{T}\tilde{\boldsymbol{L}}\right]\eta(t) + \left(\tau(t) - h\right)\eta^{\mathrm{T}}(t)\boldsymbol{M}\boldsymbol{R}_{1}^{-1}\boldsymbol{M}^{\mathrm{T}}\eta(t) + \left(h_{0} - \tau(t)\right)\eta^{\mathrm{T}}(t)\boldsymbol{N}\boldsymbol{R}_{1}^{-1}\boldsymbol{N}^{\mathrm{T}}\eta(t)\right\}$$
(21)

where Π_1 , \mathcal{A} , Λ_1 , \tilde{L} are defined in Theorem 1.

One can easily see that (11) corresponding to i = 1, j = 1, 2 is a sufficient condition to guarantee $\mathcal{E}\{\mathcal{L}V(\zeta_t) + e^{T}(t)e(t) - \gamma^2 \omega^{T}(t)\omega(t)\} < 0$ by using the Schur complement and Lemma 2.

Case 2: For $t \in \mathcal{F}_2$: Using a similar method to the Case 1, we can obtain the same conclusion, that is $\mathcal{E}\{\mathcal{L}V(\zeta_t) + e^{T}(t)e(t) - \gamma^2 \omega^{T}(t)\omega(t)\} < 0$ under the condition (11) corresponding to i = 2, j = 1, 2 for any $t \in \mathcal{F}_2$.

From the above discussions, we can conclude that

$$\mathcal{E}\left\{\mathcal{L}V(\zeta_t) + \mathbf{e}^{\mathrm{T}}(t)\mathbf{e}(t) - \gamma^2 \boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t)\right\} < 0 \qquad (22)$$

for all $t \in \mathbb{R}^+$ if the inequality (11) holds.

Under the zero initial condition, integrating both sides of (22) from t_0 to t and letting $t \to \infty$, we have

$$\mathcal{E}\left\{\| \mathbf{e}(t)\|_{2}\right\} \le \gamma \|\boldsymbol{\omega}(t)\|_{2} \tag{23}$$

Next, we consider the mean-square asymptotic stability of augmented system (5). When $\omega(t) \equiv 0$ combining with (11) and (23), we have $\mathcal{E}\{\mathcal{L}V(\zeta_i)\} < 0$, which gives $\mathcal{E}\{\mathcal{L}V(\zeta_i)\} < -\varepsilon \|\zeta(t)\|^2$ for a sufficiently small $\varepsilon > 0$, and ensures the mean-square asymptotic stability of system (5) with time-varying delay that satisfies (4). This completes the proof.

Remark 3: A less conservative result is obtained by using the convex property of matrix inequality and the idea of DCP in [28]. One can further extend the DCP method to achieve much less conservative results by dividing the delay interval into several equal subintervals by using the similar method. For the sake of technical simplicity, here we only take two equal intervals.

Based on Theorem 1, we are in a position to derive a criterion for the filter design.

Theorem 2: for given scalars \underline{h} , \overline{h} and γ , if there exist matrices $P_1 > 0$, $\overline{P}_3 > 0$, $P_i > 0$, $\overline{P}_i > 0$, (i = 1, 2), P_2 , \overline{A}_f , \overline{B}_f , \overline{C}_f , \hat{M} , \hat{N} , \hat{S} and \hat{T} with appropriate dimensions, such that

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the following LMIs hold, then the system (5) is mean-square asymptotically stable with H_{∞} norm bound γ

$$\begin{bmatrix} \tilde{\Pi}_{i} + \hat{\Lambda}_{i} + \hat{\Lambda}_{i}^{\mathrm{T}} & * & * & * \\ \mathcal{R}\tilde{\mathcal{A}} & -\mathcal{R} & * & * \\ \hat{L} & 0 & -I & * \\ \hat{\Upsilon}_{ij} & 0 & 0 & -\mathbf{R}_{i} \end{bmatrix} < 0 \ (i, \ j = 1, \ 2)$$

$$(24)$$

$$\boldsymbol{P}_1 - \bar{\boldsymbol{P}}_3 > 0 \tag{25}$$

Moreover, the parameters of H_{∞} filter in (3) are given by

$$A_{\rm f} = \bar{A}_{\rm f} \bar{P}_3^{-1}, \ B_{\rm f} = \bar{B}_{\rm f}, \ C_{\rm f} = \bar{C}_{\rm f} \bar{P}_3^{-1}$$
 (26)

where (see equations at the bottom of the page)

$$\begin{split} \tilde{\Pi}_{11} &= \begin{bmatrix} \tilde{\Pi}_{11}^{11} & * \\ \tilde{\Pi}_{21}^{21} & \tilde{\Pi}_{21}^{22} \end{bmatrix} \\ \tilde{\Pi}_{11}^{11} &= P_1 A + A^T P_1 + Q_0 + Q_1 + Q_2 - R_0 \\ \tilde{\Pi}_{11}^{21} &= \bar{P}_3 A + \bar{A}_f^T, \quad \tilde{\Pi}_{12}^{22} &= \bar{A}_f + \bar{A}_f^T \\ \tilde{\Pi}_{31} &= \begin{bmatrix} C^T \bar{\Xi}^T \bar{B}_f^T - N_{11}^T - M_{11}^T A^T \bar{P}_3^T + C^T \bar{\Xi}^T \bar{B}_f^T + N_{12}^T - M_{12}^T \end{bmatrix} \\ \tilde{\Pi}_{21} &= \begin{bmatrix} R_0 + M_{11}^T & M_{12}^T \end{bmatrix} \quad A = \begin{bmatrix} A & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \tilde{\Pi}_{51} &= \begin{bmatrix} B^T P_1 & B^T \bar{P}_3 \end{bmatrix}, \quad \hat{L} = \begin{bmatrix} L - \bar{C}_f & 0 & 0 & 0 & 0 \end{bmatrix} \\ \tilde{\Pi}_{51} &= \begin{bmatrix} 0 \hat{M} \hat{N} - \hat{M} - \hat{N} & 0 \end{bmatrix} \\ \hat{\Lambda}_1 &= \begin{bmatrix} 0 \hat{M} \hat{N} - \hat{M} - \hat{N} & 0 \end{bmatrix} \\ \hat{\Lambda}_2 &= \begin{bmatrix} 0 & 0 \hat{S} - \hat{T} \hat{T} & -\hat{S} & 0 \end{bmatrix} \\ \hat{M} &= \begin{bmatrix} M_{11}^T & M_{12}^T & M_2^T & M_3^T & M_4^T & M_5^T & M_6^T \end{bmatrix}^T \\ \hat{N} &= \begin{bmatrix} N_{11}^T & N_{12}^T & N_2^T & N_3^T & N_4^T & N_5^T & N_6^T \end{bmatrix}^T \\ \hat{S} &= \begin{bmatrix} S_{11}^T & S_{12}^T & S_2^T & S_3^T & S_4^T & S_5^T & S_6^T \end{bmatrix}^T \\ \hat{T} &= \begin{bmatrix} T_{11}^T & T_{12}^T & T_2^T & T_3^T & T_4^T & T_5^T & T_6^T \end{bmatrix}^T \end{split}$$

Proof: Defining

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{P}_1 & * \\ \boldsymbol{P}_2 & \boldsymbol{P}_3 \end{bmatrix} > 0, \quad \boldsymbol{J} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{P}_2^{\mathrm{T}} \boldsymbol{P}_3^{-1} \end{bmatrix}$$

and

$$\bar{\boldsymbol{A}}_{\mathrm{f}} = \boldsymbol{P}_{2}^{\mathrm{T}} \boldsymbol{A}_{\mathrm{f}} \boldsymbol{P}_{2}, \quad \bar{\boldsymbol{B}}_{\mathrm{f}} = \boldsymbol{P}_{2}^{\mathrm{T}} \boldsymbol{B}_{\mathrm{f}}, \quad \bar{\boldsymbol{C}}_{\mathrm{f}} = \boldsymbol{C}_{\mathrm{f}} \boldsymbol{P}_{3}^{-1} \boldsymbol{P}_{2}$$
$$\bar{\boldsymbol{P}}_{3} = \boldsymbol{P}_{2}^{\mathrm{T}} \boldsymbol{P}_{3}^{-1} \boldsymbol{P}_{2}$$

Then $P_1 > 0$ and $P_1 - \bar{P}_3 > 0$ if P > 0. Multiplying both sides of (11) with $\{J, I, I, I, I, I, I, I\}$ and its transpose. It can be shown that (24) is equivalent to (11), where

$$\begin{bmatrix} \boldsymbol{M}_{11}^{\mathrm{T}} & \boldsymbol{M}_{12}^{\mathrm{T}} \end{bmatrix} = \boldsymbol{M}_{1}^{\mathrm{T}} \boldsymbol{J}^{\mathrm{T}}, \quad \begin{bmatrix} \boldsymbol{N}_{11}^{\mathrm{T}} & \boldsymbol{N}_{12}^{\mathrm{T}} \end{bmatrix} = \boldsymbol{N}_{1}^{\mathrm{T}} \boldsymbol{J}^{\mathrm{T}}$$

Then the system (5) is mean-square asymptotically stable, with the parameters as

$$A_{\rm f} = P_2^{-{\rm T}} \bar{A}_{\rm f} \bar{P}_3^{-1} P_2^{\rm T}, \quad B_{\rm f} = P_2^{-{\rm T}} \bar{B}_{\rm f}, \quad C_{\rm f} = \bar{C}_{\rm f} \bar{P}_3^{-1} P_2^{\rm T}$$
(27)

It is algebraically equivalent [29] with $A_{\rm f} = \bar{A}_{\rm f} \bar{P}_3^{-1}$, $B_{\rm f} = \bar{B}_{\rm f}$, $C_{\rm f} = \bar{C}_{\rm f} \bar{P}_3^{-1}$. This completes the proof.

4 Numerical examples

This section aims to demonstrate that the method proposed above not only with less conservativeness comparing with the existed ones, but also with perfect reliability.

Example 1: Consider the linear system (1) with the following parameters [30]

$$A = \begin{bmatrix} 0 & 3 \\ -4 & -5 \end{bmatrix}, B = \begin{bmatrix} -0.5 \\ 0.9 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \end{bmatrix}, L = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

Assume $\tau(t)$ in (4) satisfies $0.2 < \tau(t) < 0.48$. To illustrate the result by using our proposed method is of less conservativeness, in this example, we assume the sensor signal without any distortion, that is $\Xi \equiv 1$. Based on Theorem 2, we can obtain $\gamma_{\min} = 0.19$, whereas $\gamma_{\min} =$

$$\tilde{\Pi}_{1} = \begin{bmatrix} \tilde{\Pi}_{11} & * & * & * & * & * & * \\ \tilde{\Pi}_{21} & -Q_{1} - R_{0} & * & * & * & * & * \\ \tilde{\Pi}_{31} & 0 & 0 & * & * & * & * \\ 0_{n \times 2n} & 0 & 0 & -Q_{0} - R_{2} / \delta & * & * & * \\ 0_{n \times 2n} & 0 & 0 & R_{2} / \delta & -Q_{2} - R_{2} / \delta & * \\ \tilde{\Pi}_{51} & 0 & 0 & 0 & 0 & -\gamma^{2} \end{bmatrix}$$

$$\tilde{\Pi}_{2} = \begin{bmatrix} \tilde{\Pi}_{11} & * & * & * & * & * \\ \tilde{\Pi}_{21} & -Q_{1} - R_{0} - R_{1} / \delta & * & * & * & * \\ \tilde{\Pi}_{31} & 0 & 0 & * & * & * \\ 0_{n \times 2n} & R_{1} / \delta & 0 & -Q_{0} - R_{1} / \delta & * & * \\ 0_{n \times 2n} & 0 & 0 & 0 & -Q_{2} & * \\ \tilde{\Pi}_{51} & 0 & 0 & 0 & 0 & -\gamma^{2} \end{bmatrix}$$



Fig. 2 Curves by using the reliable filtering parameters



Fig. 3 Curves by using the reliable filtering parameters

1.42 by using the method of Theorem 1 in [30]. Obviously, our result is much better than that ones in [30]. *Example 2:* Consider the controlled process (1) with the following parameters

$$A = \begin{bmatrix} 0 & -1 & -0.5\\ 1 & -2 & 1\\ 0 & -0.5 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} -2\\ 1\\ -0.5 \end{bmatrix}$$
$$C = \begin{bmatrix} -10 & 0 & 0\\ 0 & 50 & 50 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0\\ 0 & 1 & 0 \end{bmatrix}$$
$$\omega(t) = \begin{cases} 1, & 5 \le t \le 10 \text{ s}\\ -1, & 15 \le t \le 20 \text{ s}\\ 0, & \text{otherwise} \end{cases}$$

Assume the network index $\tau(t)$ satisfies $0.1 < \tau(t) < 0.48$, and the sensor signal is subject to probabilistic sensor distortion with $\mu_1 = 0.6$.

Apply the LMI toolbox, a set of feasible solutions to LMIs (24) with the performance level $\gamma = 5.0$ and distortion level $\mu_1 = 0.6$ can be obtained. Furthermore, the filter parameters



Fig. 4 Curves by using the normal filtering parameters

are computed by (26) as follows

$$A_{\rm f} = \begin{bmatrix} -4.8834 & -0.5386 & -0.7000 \\ 2.0471 & -2.7664 & 1.0882 \\ -2.0329 & -1.5562 & -1.8624 \end{bmatrix}$$

$$B_{\rm f} = \begin{bmatrix} 0.2519 & 0.0039 \\ 0.0238 & -0.0117 \\ -0.0287 & -0.0051 \end{bmatrix}$$

$$C_{\rm f} = \begin{bmatrix} -2.5990 & -0.6188 & -0.2964 \\ -0.6329 & -0.5406 & 0.0435 \end{bmatrix}$$
(28)

By using the parameters given in (28), we can obtain the simulation curves of z(t) and $z_4(t)$ under the case of the sensor distortion occurring in the first channel (see Figs. 2 and 3), whereas Figs. 4 and 5 are curves of the system under the sensor distortion using the normal filtering parameters, that is the parameters are calculated by Theorem 2 by letting $\mu_1 = 1$. Comparing with those two sets of figures, one can obviously see that the designed filter produces a better estimate of z(t) under the sensor signal distortion.



Fig. 5 Curves by using the normal filtering parameters

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5 Conclusion

In this paper, we addressed the problem of the reliable H_{∞} filter design for sampled-data systems with a consideration of probabilistic sensor distortion. To obtain a less conservative result, an improved DCP method is proposed by using the convexity property of the matrix inequality. LMI-based conditions are formulated for the existence of admissible filters, which ensure the filtering-error systems are mean-square asymptotically stable with prescribed H_{∞} disturbance attenuation level. Two illustrative examples are exploited to show the effectiveness of the proposed design procedures.

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